WORKING PAPER · NO. 2020-31

R < 1 as an Economic Constraint: Can We "Expand the Frontier" in the Fight Against Covid-19?

Eric Budish

APRIL 2020

Becker Friedman Institute
FOR ECONOMICS AT UCHICAGO

5757 S. University Ave.
Chicago, IL 60637
Main: 773.702.5599
bfi.uchicago.edu

$R < 1$ as an Economic Constraint: Can We "Expand the Frontier" in the Fight Against Covid-19?

Eric Budish*†

April 1, 2020

Abstract

This note suggests that we view $R < 1$ as an economic constraint, allowing social welfare in the traditional sense (economic activity, societal well-being) to be the policy objective. This formulation highlights two key questions at the intersection of health and economics research in response to the Covid-19 crisis. First, what activities maximize social welfare subject to the constraint that disease-transmission is contained, i.e., $R < 1$. Second, what are ways to "expand the frontier" of how much social welfare we can achieve while keeping disease-transmission contained. For example, could widespread use of masks and gloves, society-wide campaigns to "wash your hands," "stay home if sick," and "don't touch your face," and aggressive testing, together allow us to meaningfully increase the level of economic activity and societal well-being that is possible while keeping $R < 1$?

*University of Chicago Booth School of Business, eric.budish@chicagobooth.edu. I am not a medical expert but rather am an economist with expertise in market design, an engineering-oriented branch of applied microeconomics. I try to be explicit throughout this note where I am using my own non-expert understanding of medical facts. I thank Ethan Che and Jia Wan for research assistance. I thank Austan Goolsbee, Ralph Koijen, Yueran Ma, Neale Mahoney, Adi Sunderam, Chad Syverson, Sarah Taubman and Heidi Williams for initial feedback. I plan to revise this note after receiving additional feedback.

†This note is dedicated to my partners in shelter-in-place and in life: Emma, Nathan and Jacob.

1 A Proposed Formulation of the Covid-19 Policy Problem

Economists usually think about public policy with a constrained maximization problem in the back of their heads, something like

$$\begin{aligned} &\max \text{Social Welfare} \\ &\quad \text{subject to} \\ &\text{Technological Constraints} \\ &\text{Incentive Constraints} \end{aligned} \tag{1}$$

where "Social Welfare" includes both the economic and non-economic dimensions of well-being, and is sometimes called "Social Utility" (or, if there is a representative agent, simply "Utility"). Constraints arise due to both Technology (we can't all have infinite of everything) and Incentives (it is hard to get someone to work hard or innovate without compensation).

Health policy experts, in their urgent response to the Covid-19 crisis, seem to have a constrained maximization problem in the back of their heads like

$$\begin{aligned} &\min \text{Spread of Covid-19} \\ &\quad \text{subject to} \\ &\text{Keeping Society Functioning} \end{aligned} \tag{2}$$

This perspective makes sense as a response to the dramatic — and frankly, terrifying — exponential growth of Covid-19 cases around the world. Unchecked, Covid-19 cases will continue to double every few days, overwhelming medical systems around the world and leading to tens of millions of deaths.

The difficulty with (2), however, is that it is impossible to think about the very real tradeoffs we are facing. The extreme versions of social distancing that are called for by (2) — closing schools, essentially shuttering entire industries (travel, restaurants, sports, entertainment, gyms, conferences), avoiding close interactions with other people — will themselves have an enormous human toll. Unemployment claims during the week of March 23rd exceeded 3 million, or more than 15 times the usual level. An influential epidemiological model discusses the possibility of childrens' schools being closed for most of the next 2 years.[1]

I suggest that the following constrained maximization problem can be a useful way of incorporating a version of (2) into (1), i.e., a version of the urgent health perspective into the usual

[1]Source: Imperial College Covid-19 Response Team, "Impact of non-pharmaceutical interventions (NPIs) to reduce COVID19 mortality and healthcare demand," March 16, 2020. Excerpt from page 11: "The right panel of Table 4 shows that social distancing (plus school and university closure, if used) need to be in force for the majority of the 2 years of the simulation."

economic perspective:

$$\begin{aligned} &\max \text{ Social Welfare} \\ &\quad \text{subject to} \\ &\text{Technological Constraints} \\ &\text{Incentive Constraints} \\ &R < 1\text{: Reduce the Covid-19 Average Transmission Rate to Below 1} \end{aligned} \tag{3}$$

This formulation superficially looks like (1) — in particular, it has the usual economic objective function, social welfare — but functionally it will do well at achieiving the medical objective in (2), because of the additional constraint that has been added: reduce "R", the average transmission rate of Covid-19, to below the critical threshold of 1.[2,3]

As is widely understood, diseases with average transmission rates strictly above 1 eventually infect huge numbers of people (until so many have been infected that there aren't enough remaining susceptible targets for the disease to further infect), whereas diseases with average transmission rates strictly below 1 dissipate. If disease transmission can be reduced to below 1, then the scale of the crisis, while not *absolutely* minimized as in formulation (2), will be *approximately* minimized.

To be concrete: as of March 29th, there have been about 2,500 Covid-19 deaths in the US and about 35,000 Covid-19 deaths globally. For perspective there will be about 40,000 traffic fatalities in the US this year and about 40,000 deaths in the US from handguns. The reason why the Covid-19 fatality rate is so terrifying is the possibility that these numbers will continue to grow exponentially. If the numbers of infections and deaths continue to grow at their current trajectory, the death toll could exceed 2,000,000 in the US this year — or more than 25 times the total burden of traffic fatalities and gun deaths — and that is without even factoring in the additional fatalities that result from an overwhelmed medical system.[4] Whereas, if we can constrain the exponential growth — that is, if R can be constrained to below 1 — then we will still have a tragedy but of less-tragic proportions.

[2]In principle this optimization problem can be formulated with the constraint $R \leq x$ for any feasible x. As discussed below, if we think dynamically about the problem there is a good argument for using $x < 1$ for a period of time in areas where the infection rate is already very high or the medical system is or will soon be overwhelmed.

[3]This approach of maximizing economic activity subject to a disease-containment constraint can be interpreted as a formalization of my colleague Austan Goolsbee's "First Law of Virus Economics," as he has discussed in various media since mid-March. A March 29th blog post by John Cochrane also discusses $R < 1$ extensively. I imagine that many other economists have some version of (3) implicitly in their thinking about the Covid-19 response. I hope this note is useful nonetheless.

[4]Imperial study, page 7 ("In the (unlikely) absence of any control measures or spontaneous changes in individual behavior, we would expect ... 2.2 million [deaths] in the US, not accounting for the potential negative effects of health systems being overwhelmed.")

Note: Is it Possible to Reduce R Below 1? In a word, yes. China has done it, with 0 new locally-transmitted cases reported on March 19th, 2020. South Korea, Singapore and Hong Kong have done it.[5]

Most estimates for the initial transmission rate of Covid-19, known as R_0, estimate R_0 of around 2.5-3.0, though there is considerable uncertainty around that estimate.[6] This initial transmission rate depends on both the properties of the virus (e.g., how it is transmitted) and behavior in the society that the virus is embedded in (e.g., how often people behave in ways that risk transmission).

The subsequent transmission rate, denoted R in the above or R_t if we want to think more dynamically, can differ from the initial transmission rate R_0 for two basic reasons. First, even with no change in societal behavior, R declines eventually once enough people have already been infected: there is less room for the virus to further spread. This is what generates the well-known S-shaped curves in SIR models,[7] where there is a phase of exponential growth and then at some point so much of society has already been infected that the growth rate of the disease transitions from $R > 1$ to $R < 1$ and eventually converges towards $R = 0$. This decline is known as herd immunity. However, given Covid-19 estimated mortality rates, achieving $R < 1$ via herd immunity would come at considerable cost in lives lost.

Second, R can be lower than R_0 because of interventions — in the case of Covid-19, which has no vaccines or treatments to date, "non-pharmaceutical interventions" or NPIs. These include various forms of social distancing, hand-washing campaigns, reminders not to touch your face, encouragement to stay home if ill, more widespread testing, etc.

Intuitively, if the initial R_0 is around 2.5 or 3.0, then if social distancing and other NPIs can reduce the spread by two-thirds (i.e., $\frac{3.0-1.0}{3.0} = \frac{2}{3}$), then we can achive $R \leq 1$. The experiences of China, South Korea, Singapore and Hong Kong all indicate that this is technologically feasible.

Please see Section 4 for additional discussion of how to reduce R.

Discussion: Uncertainty, Dynamics, and Medical System Capacity. Should We Initially Overshoot $R < 1$? There is uncertainty about the impact the menu of NPIs can have on R. There is also uncertainty about the impact that the endogenous response of people who

[5]See the Financial Times coronavirus case trajectories by country, available at https://www.ft.com/coronavirus-latest. Please see Atul Gawande, "Keeping the Coronavirus from Infecting Health-Care Workers," on some of the medical lessons learned from Singapore and Hong Kong.

[6]The Imperial College team uses R_0 of 2.4 and a range of 2.0-2.6 in their paper circulated March 16th 2020. In testimony on March 25th Neil Ferguson, the PI of the Imperial College team, reports that they have revised their estimates to "on the order of 3 or a little bit above rather than the 2.5 level", based on the disease's spread in Europe. James H. Stock (2020) considers R_0 of about 3.5 without interventions and discusses the various sources of uncertainty.

[7]Please see Atkeson (2020) and Stock (2020) for introductions to SIR models written for economists. If one conceptualizes (3) with the constraint $R \leq x$ with $x > 1$, then one has to take into account the more-complicated SIR dynamics as the population of infected individuals grows. If $R < 1$ then the SIR dynamics are much simpler.

become aware of (and scared of) the disease, and therefore behave differently (go outside less, wash hands more), will have on R.

Given this uncertainty it may be socially optimal to at first overshoot and aim for R considerably less than 1, and then to gradually relax the constraint towards $R < 1$ as more information accumulates. The case for such overshooting is enhanced if the current level of infections is already so high that (a) the medical system is overwhelmed, or (b) the disease burden even if we achieve $R \approx 1$ from here forward is already so high from a societal perspective that (3) does not satisfactorily approximate the objective in (2).

Thus, thinking dynamically, the reader may wish to interpret program (3) as first being implemented using $R \leq x$ for some $x < 1$, and then transitioning to a steady state of $R \leq 1$. This interpretation corresponds to the idea of the "The Hammer and the Dance" in an influential Medium post of Pueyo (2020).

2 Two Advantages of this Formulation of the Problem

There are two concrete benefits of formulation (3).

2.1 Tradeoffs.

First, (3) allows us to think about *tradeoffs.* We can evaluate actions by their ratio of benefits to costs — the ratio of Social Welfare benefits (economic activity, societal functioning, etc.) against Disease-Transmission costs.

While my formulation of (3) above is purposefully informal, economists will recognize that the new constraint $R < 1$ places a shadow cost on activities that increase disease-transmission. And, conversely, the constraint places a shadow benefit on activities that reduce disease-transmission, because such activities free up capacity to do other activities with high social-welfare benefits but some increase in disease-transmission.

A more formal presentation of (3) is contained in the last section of this note for those readers who find this useful. The formal presentation provides an explicit formula for the tradeoff of benefits against both traditional economic costs and the shadow cost from disease transmission, and clarifies what parameters need to be understood to incorporate the health objective into traditional social welfare optimization.

2.2 Expanding the Frontier.

Second, this formulation highlights the gains from expanding what economists call the "Production Possibilities Frontier" — *how much social welfare can be achieved for a given level of disease-transmission risk.* If we can expand the frontier, then we can create more economic activity and societal well-being while still keeping R below 1.

The formal presentation clarifies what expansions of the frontier are especially valuable: technologies that reduce the level of disease-transmission risk for activities with high social-welfare benefits, without adding too much additional cost to these activities.

3 Two Key Policy Questions

These two economic concepts — tradeoffs, and expanding the frontier — in turn suggest two questions that should be front and center for research.

3.1 *Question 1: How do we maximize economic activity and societal well-being while keeping $R < 1$?*

The notion of economic tradeoffs and maximizing social welfare per unit of disease-transmission makes it obvious why we should allow essential activities even though they come with non-zero transmission risk. It also makes it obvious why a temporary shutdown of crowded bars and certain mass-gatherings makes sense: some activities have too much disease-transmission risk per unit of societal benefit.

3.2 *Question 2: How do we expand the frontier of how much economic activity and societal well-being we can achieve while keeping $R < 1$?*

The notion of "expanding the frontier" makes clear why society-wide hand-washing campaigns, society-wide education about the importance of not touching one's face with unwashed hands ("Don't touch your face"), ensuring the sick stay home, encouraging physical distance where possible, putting paper towels and garbage bins in the right place in bathrooms, putting tissue boxes near elevators and widely-used door-handles, encouraging those who feel even slightly ill to stay home, etc., are win-win, both for the economy and society. They reduce disease-transmission at relatively little cost.

Similarly, once testing is more widely available it can meaningfully expand the frontier, for two reasons. First, especially in conjunction with contact-tracing, testing can help ensure that those currently infected who are either asymptomatic or only mildly symptomatic stay isolated. Second, testing can allow those who have recovered and have antibodies to more fully engage in the economy.

4 Could Masks, Gloves, "Wash Your Hands," "Stay Home if Sick," "Don't Touch Your Face," and Widespread Testing Together Save the Economy?

This section gives an example of the kind of thinking I have in mind regarding expanding the production possibilities frontier. Please note that it is based on my non-expert understanding of the early medical facts about the spread of Covid-19 and thus should be regarded as hypothetical.

My non-expert understanding of the early scientific reports on Covid-19 is:[8]

- The R_0 of Covid-19 without any policy interventions (either pharmaceutical or non-pharmaceutical) is estimated to be around 2.5-3.0, though there is considerable uncertainty (see fn. 6)

- The virus spreads primarily through (a) droplets from an infected person who coughs or sneezes reaching a susceptible's nose or mouth, or (b) a susceptible touching a surface that was coughed on or sneezed on by an infected person, or that was touched by an infected person with virus on their hand (from a cough, sneeze, or from touching their mouth or nose). The extent of airborne transmission (e.g., in small confined spaces like elevators) is an important unknown.

- Infected people can spread the virus for a period of time without knowing they are infected. Most likely, they have to cough, sneeze, or touch a surface after touching their mouth or nose. An important unknown is the extent to which the virus can spread through prolonged close contact (e.g., face-to-face conversation), even without coughs or sneezes.

- Infected people are less likely to spread the virus if they are wearing a mask (because their droplets are caught) and protective gloves (because they are less likely to contaminate a surface). Please note that these masks are not the N95 masks that are in such scarce supply

[8] This non-expert understanding is based on a variety of sources including the CDC and Johns Hopkins web sites on how the virus spreads, Li et al (2020) on asymptomatic spread, Doremalen et al (2020) on aerosol and surface stability, Gawande (2020) on medical lessons learned from Singapore and Hong Kong, and numerous others.

for frontline medical workers. The purpose of the masks is to reduce the amount of virus the wearer spreads, not to protect the wearer from infection.

- Susceptible people are less likely to get the virus if they are wearing protective gloves – both because they are less likely to pick up virus from a contaminated surface and because they are less likely to touch their face.

Given these facts, hypothetically suppose that some variation on the following set of policies can reduce R to below 1:

1. Widespread public-education campaign about the importance of hand washing and not touching your face with unwashed hands.
2. Mandatory home-quarantine for anybody with potential virus symptoms (e.g., fever or persistent cough), complemented with a significant increase in testing capacity and contact tracing.[9]
3. Encourage social distancing practices (e.g., six feet of distance) wherever possible.
4. Mandatory wearing of masks in many situations outside the home — e.g., all contexts where it is unavoidable to come within six feet of others for a non-trivial period of time.
5. Mandatory wearing of gloves in many situations outside the home — e.g., all contexts where it is unavoidable to touch surfaces that strangers will also touch.

If these policies together could reduce R to below 1, then we could have a mostly-functioning economy and society, albeit with everybody wearing masks and gloves for a while (specifically, until either the disease is eradicated or there is a successful treatment or vaccine). There would still be certain specific parts of the economy that might have to remain shuttered, such as restaurants and bars (it is hard to eat and drink with a mask on), but much of the economy could function with some normalcy.

Again, this list is hypothetical. There is no evidence to suggest that this particular combination of policies and interventions is enough to bring R below 1. My point is that we should urgently try to figure out whether some combination of policies like the above could succeed — we should try to be creative about combinations of policies and interventions that together bring R below 1, without an indefinite period of significant harm to the economy and society. If R_0 were 10 this would seem helpless but with R_0 on the order of 2.5-3, with a relatively clear understanding of how the disease does and doesn't spread, and with several empirical examples to date of countries bringing their $R < 1$, it seems achievable.

[9]Enforcing this policy would require both sticks and carrots, with the latter including widely-available and generous paid sick leave.

5 Conclusion: A Plea for Research.

Whether some set of policies like the set discussed above could reduce R to below 1 is of course a question for medical experts and not economists. My point is to suggest a way of thinking about the problem.

The two key questions are: How do we maximize economic and societal activity while keeping the disease contained — i.e., while keeping $R < 1$? How do we expand the frontier of how much economic and societal activity is possible while keeping $R < 1$?

Some policies and interventions seem like the proverbial free lunch: they expand the frontier without that much cost. These include massive public campaigns to encourage hand washing, to educate the public about the importance of not touching your face, placing hand sanitizer in heavily trafficked places, encouraging the even-slightly sick to stay home, more widespread testing, etc.

We should also think creatively about interventions that lower R meaningfully relative to the social welfare gains they enable, even if the interventions themselves are not costless. The idea of widespread use of masks and gloves, as discussed in Section 4, is in this spirit. Contact tracing is another example.

If there is an idea in this note that is not conceptually obvious, it is the notion of expanding the production possibilities frontier in the battle against Covid-19. Expanding the production possibilities frontier is a natural concept to economists, but perhaps not to medical experts. Whereas the fact that there exists scope to expand the frontier in this context — R is *not* a constant of the virus, but depends on both the properties of the virus and the behavior of the society it is embedded in — is likely obvious to medical experts but may not be to economists.

My sincere hope is that medical experts and economists can work together to engineer creative ways to reduce R, and enable the economy and society to return to some semblance of normalcy.

References

Atkeson, Andrew G., "What will be the economic impact of COVID-19 in the US? Rough estimates of disease scenarios," 2020. March 17. Federal Reserve Bank of Minneapolis Staff Report No. 595.

Centers for Disease Control and Prevention, "How coronavirus spreads," 2020. March 4. Accessed on March 31, 2020 from `https://www.cdc.gov/coronavirus/2019-ncov/prevent-getting-sick/how-covid-spreads.html`.

Cochrane, John H., "Beyond testing — The central question for pandemic policy," 2020. March 29. `https://johnhcochrane.blogspot.com/2020/03/beyond-testing-central-question-for.html`.

Gawande, Atul, "Keeping the coronavirus from infecting health-care workers," The New Yorker 2020. March 21. `https://www.newyorker.com/news/news-desk/keeping-the-coronavirus-from-infecting-health-care-workers`.

Imperial College COVID-19 Response Team (Ferguson, Neil M et al.), "Impact of non-pharmaceutical interventions (NPIs) to reduce COVID19 mortality and healthcare demand," 2020. March 16. Working paper.

Johns Hopkins Medicine, "Coronavirus at a glance," 2020. Accessed on March 31, 2020 from `https://www.hopkinsmedicine.org/health/conditions-and-diseases/coronavirus/coronavirus-facts-infographic`.

Li, Ruiyun, Sen Pei, and Bin Chen et al., "Substantial undocumented infection facilitates the rapid dissemination of novel coronavirus (SARS-CoV2)," *Science*, 2020. March 16.

Pueyo, Tomas, "Coronavirus: The hammer and the dance," 2020. March 19. `https://medium.com/@tomaspueyo/coronavirus-the-hammer-and-the-dance-be9337092b56`.

Stock, James H., "Coronavirus data gaps and the policy response to the novel coronavirus," 2020. March 23. Working paper.

van Doremalen, Neeltje, Trenton Bushmaker, and Dylan H. Morris et al., "Aerosol and surface stability of SARS-CoV-2 as compared with SARS-CoV-1," *New England Journal of Medicine*, 2020. March 17.

A Appendix: Formal Versions of Formulations (1)-(3)

We can express the main theoretical ideas in this note with a very stripped down version of a social planner problem. There is a single representative agent who chooses a vector of actions, denoted $a \in A \subset \mathbb{R}^n$ with each action $a_i \in [0, \bar{a}_i]$, to maximize their utility. Utility from action vector a is denoted $U(a) \equiv v(a) - c(a)$, where $v : A \to \mathbb{R}$ is a strictly concave, continuously differentiable function that describes the benefits associated with the vector of actions, and $c : A \to \mathbb{R}$ is a strictly convex, continuously differentiable function that describes the costs associated with the vector of actions. Normalize $v(0) = c(0) = 0$ and assume that $U(a)$ has a maximum in the interior of A. We have in mind that actions are things that the representative agent wants to do at least some of, in a value net of costs sense, such as work, go to school, socialize, go to stores, etc.

The role of the policymaker is to choose which actions are permitted in society. In our initial formulation of the problem, we exclude all of the usual reasons that the policymaker's interests might diverge from the outcome of agents' self-interested behavior: externalities, public goods, social insurance, etc. Instead, at first, we assume the policymaker's interests and the representative agent's interests are perfectly aligned. The policymaker chooses a subset $A' \subseteq A$ of actions that are permissible, and the agent then optimizes. We can express the optimization problem as:

$$\begin{aligned} \max_{A' \subseteq A} \quad & U(a) \\ \text{subject to} \quad & a \in A' \\ & U(a) \geq U(a'), \forall a, a' \in A' \end{aligned} \tag{1}$$

Since the policymaker's interests align with the representative agent's interests, the policymaker will optimally choose $A' = A$ and this reduces to a single-agent optimization problem, $\max_{a \in A} U(a)$. The agent will choose a^* such that, for each action a_i in the action vector, we have

$$\frac{\partial v(a^*)}{\partial a_i} = \frac{\partial c(a^*)}{\partial a_i},$$

i.e., marginal benefits of the action equal marginal costs of the action.

Now suppose that the agent's actions affect disease-transmissibility. Formally, there is a weakly increasing, linear function $R : A \to \mathbb{R}$ that denotes the average disease-transmissibility as a linear function of the representative agent's actions. In principle, we can think of mandatory social distancing policies either as eliminating some actions from the agent's choice set A or as reducing their payoff (e.g., threat of punishment). For simplicity, here I will think of such policies in terms of direct restrictions on the agent's choice set.

The pure medical objective can be expressed as

$$\begin{aligned} \min_{A' \subseteq A} \quad & R(a) \\ \text{subject to} \quad & a \in A' \\ & U(a) \geq U(a'), \forall a, a' \in A' \end{aligned} \tag{2}$$

Notice that whereas in Formulation (1) the social planner and representative agent's interests were exactly aligned, in Formulation (2) they are not. In Formulation (2) the representative agent chooses the actions that maximize her utility, whereas the social planner's objective is to minimize disease-transmissibility. If the disease were sufficiently lethal / negative for utility — e.g., if utility from getting the disease were $-\infty$ — then preferences would align again. But as long as the representative agent's disutility from getting the disease is not infinite, they will make choices that misalign with a pure $\min R(a)$ objective function.

Note as well that Formulation (2) could be enhanced to incorporate a "keep society functioning" constraint, for example through a minimum utility threshold. There would still be divergence of interests between the utility-maximizing agent and the social planner's objective.

My proposed formulation of the problem is:

$$\begin{aligned} \max_{A' \subseteq A} \quad & U(a) \\ \text{subject to} \quad & a \in A' \\ & U(a) \geq U(a'), \forall a, a' \in A' \\ & R(a) \leq 1 \end{aligned} \tag{3}$$

As in Formulation (1) the social planner's interests align with the representative agent's interests, but with the crucial exception that the planner faces a disease-transmissibility constraint that the agent does not necessarily internalize. Therefore, instead of allowing the agent to choose the utility-maximizing vector within the entire set of actions, the social planner will limit the agent to the set of actions for which $R(a) \leq 1$. Denote by r_i the linear effect action a_i has on disease-transmissibility, and let r denote the vector of such weights. The set of actions consistent with $R(a) \leq 1$ is thus $A_{R\leq 1} = \{a : \langle r, a \rangle \leq 1\}$, where the notation $\langle r, a \rangle$ denotes the vector product $\sum_{i=1}^{n} a_i r_i$.

The agent maximizes utility within this set, so the problem reduces to a single-agent constrained optimization problem, $\max_{a \in A} U(a)$ subject to $\langle r, a \rangle \leq 1$.

The Lagrangian of this problem is

$$\mathcal{L}(a,\lambda) = U(a) - \lambda[\langle r, a\rangle - 1]$$

with corresponding KKT conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}(a^*)}{\partial a_i} = \frac{\partial U(a^*)}{\partial a_i} - \lambda r_i = 0,\ \forall i \\
\lambda \geq 0 \\
\langle r, a\rangle \leq 1 \\
\lambda[\langle r, a\rangle - 1] = 0.
\end{aligned}$$

These KKT conditions imply that at the optimum action a^* we have:

$$\frac{\partial v(a^*)}{\partial a_i} = \frac{\partial c(a^*)}{\partial a_i} + \lambda r_i \tag{4}$$

or

$$a_i = 0 \text{ if } \left.\frac{\partial v(a^*)}{\partial a_i}\right|_{a_i=0} < \left.\frac{\partial c(a^*)}{\partial a_i}\right|_{a_i=0} + \lambda r_i$$

or

$$a_i = \bar{a}_i \text{ if } \left.\frac{\partial v(a^*)}{\partial a_i}\right|_{a_i=\bar{a}_i} > \left.\frac{\partial c(a^*)}{\partial a_i}\right|_{a_i=\bar{a}_i} + \lambda r_i$$

The difference versus the original optimization problem is that the agent incorporates the shadow cost of the action's effect on disease-transmissibility, λr_i. If this shadow cost is sufficiently high for an action relative to its benefits less costs, then the agent optimally chooses 0 of the action. Conversely, if the benefit of the action is sufficiently high relative to these shadow costs, the agent may optimally choose the maximum $\bar{a}_i$ of the action.

The optimization condition (4) implies that for actions $a_i \in (0, \bar{a}_i)$ and $a_j \in (0, \bar{a}_j)$ in the optimal action vector, with both $r_i, r_j > 0$, the following holds:

$$\frac{\frac{\partial v(a^*)}{\partial a_i} - \frac{\partial c(a^*)}{\partial a_i}}{r_i} = \frac{\frac{\partial v(a^*)}{\partial a_j} - \frac{\partial c(a^*)}{\partial a_j}}{r_j}. \tag{5}$$

In words, condition (5) says that actions will be chosen such that the ratio of marginal social value to disease transmission is equalized across actions. The r vector can be interpreted as a vector of "prices" where the prices are the marginal effects actions have on disease-transmission. Actions that are high benefit-to-cost in the traditional sense will still be chosen even if they come with some increase in disease-transmissibility, whereas actions with high enough disease-transmissibility will be avoided even if they have benefits that exceed costs in the traditional sense.

Now suppose that the social planner can make some intervention, parameterized by t, which reduces the average disease transmission of particular actions ("expands the frontier"). For example: encouraging hand-washing, educating the public about not touching one's face with unwashed hands, mandating the sick stay home, masks, gloves, etc. Formally, we assume $\frac{dr_i(t)}{dt} \leq 0, \forall i$. In words: an increase in the level of intervention t always at least weakly decreases the disease transmission of all actions (per unit of the action). We also allow that the intervention weakly increases the costs of some actions; we now denote the cost vector by $c(a,t)$.

Given level of intervention t, the constrained optimization problem is now $\max_{a\in A} U(a)$ subject to $\langle r(t), a\rangle \leq 1$. The Lagrangian of this problem is $\mathcal{L}(a,\lambda,t) = v(a) - c(a,t) - \lambda[\langle r(t), a\rangle - 1]$. Let $V(t)$ denote the level of social welfare at the optimum given t. By the envelope theorem we have:

$$\frac{dV(t)}{dt} = -\frac{\partial c(a^*(t),t)}{\partial t} + \lambda(t)\left(\sum_i -\frac{dr_i(t)}{dt} a_i^*(t)\right) \tag{6}$$

In words, (6) tells us that interventions that reduce disease-transmission are especially valuable at the margin if:

- They reduce the *level* of disease-transmission risk a lot (i.e., $-\frac{dr_i(t)}{dt}$ is high) ...
- of actions that society does a lot of (i.e., $a_i^*(t)$ is high), because they are of high social value relative to transmissibility costs ...
- and the intervention per se is not too harmful to utility (i.e., $\frac{\partial c(a^*(t),t)}{\partial t}$ is low).

Thus, (6) might help guide what non-pharmaceutical interventions are especially beneficial to social welfare.